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Technical Notes

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Inverse Radiation Problem in Axisymmetric Free Flames

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Introduction

THE measurement of flame temperature is an important subject in combustion analysis. Optical measurement approaches have been especially favored because of minimal disturbance of the medium being probed. A lot of work has been reported on the reconstruction of flame temperature fields by infrared spectral measurements. Zhang et al.,[1] Yousefian and Lallemand,[2] and Yousefian et al.[3] have determined the temperature and species concentration profiles of axisymmetric flame by low-resolution emission and transmission infrared spectral measurements. Low resolution of spectral measurements ensures the simplicity of the instrument but complicates the inversion problem because the measurements are averaged values over a spectral interval of corresponding monochromatic values. Solomon et al.[4] and Best et al.[5] have reconstructed the species and temperature profiles of sooting diffusion flames based on Fourier-transform infrared emission and transmission spectral data by iteration method. Hall and Bonczyk[6] have used the algorithms of Fourier transform tomography to reconstruct local soot absorption coefficient and temperature of sooting flames. All four of these works are based on optical thin flames. This confines the application of their method.

In the condition that the media radiative properties, such as absorption coefficient, are given, Li and Ozisik,[7] Siewert,[8,9] and Li[10] have reconstructed the temperature profiles or source terms in semitransparent media from the emission data by conjugate gradient method.

The purpose of this Note is to extend the optimization technic of parameters estimation presented in Ref. 7 to reconstruct temperature and absorption coefficient profiles for an axisymmetric free flame from the knowledge of the outgoing monochromatic emission and transmission radiation intensities. The source term and absorption coefficient profiles are expressed as polynomial-type functions of space ordinate. The inverse problems are formulated as optimization problems and solved by using the conjugate gradient method. The effects of the measurement errors on the accuracy of the inverse analysis will be examined.

Direct Problem

As illustrated in Fig. 1, we considered an absorbing, emitting, nonscattering, axisymmetric medium with transparent boundary. The refractive index of medium is assumed to be unity compared with the surroundings. A beam of infrared laser is projected into the medium. The line of sight between laser and detector is orthogonal to the symmetry axis of the flame. The direct problem concerned here is to find the outgoing radiation intensities for the known temperature and absorption coefficient profiles. For the axisymmetric medium at local thermodynamic equilibrium, in each cross section the outgoing spectral radiation intensity $I_{1\lambda}(y)$ can be expressed as

$$I_{1\lambda}(y) = \int_{x_0(y)}^{x_1(y)} \kappa_\lambda(r) S_\lambda(r) \exp\left[-\int_x^{x_1(y)} \kappa_\lambda(r)\,\mathrm{d}x'\right]\mathrm{d}x + I_{0\lambda}\exp\left[-\int_{x_0(y)}^{x_1(y)} \kappa_\lambda(r)\,\mathrm{d}x\right] \tag{1}$$

where κ_λ is the spectral absorption coefficient, $I_{0\lambda}$ is the spectral radiation intensity of infrared laser, and $x_0(y) = -\sqrt{(R^2 - y^2)}$ and $x_1(y) = \sqrt{(R^2 - y^2)}$ are the integral lower and upper bounds, respectively. The source term S_λ is related to the temperature T in the medium by

$$S_\lambda = \frac{2C_1}{\lambda^5[\exp(C_2/\lambda T) - 1]} \tag{2}$$

where C_1 and C_2 are blackbody radiation constants. If the laser is turned off, the outgoing emission radiation intensity $I_{2\lambda}(y)$ can be expressed as

$$I_{2\lambda}(y) = \int_{x_0(y)}^{x_1(y)} \kappa_\lambda(r) S_\lambda(r) \exp\left[-\int_x^{x_1(y)} \kappa_\lambda(r)\,\mathrm{d}x'\right]\mathrm{d}x \tag{3}$$

When T and κ_λ are known, the numerical solution of the direct problem can be obtained by the trapezoidal numerical quadrature. In this approach, in order to ensure the accuracy of solution the length of line of sight in the media is divided into 100 sections.

Inverse Problem

In the inverse analysis the temperature and absorption coefficient profiles are estimated by using the measured data of outgoing

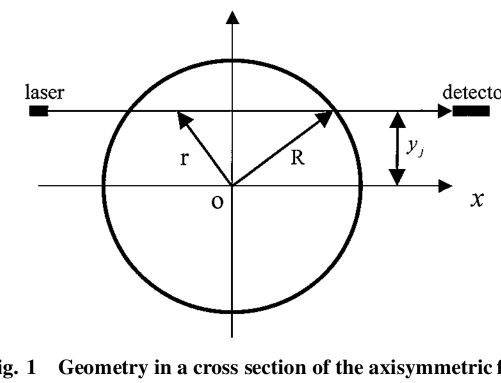

Fig. 1 Geometry in a cross section of the axisymmetric flame.

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spectral radiation intensities. The transmittance $\tau_\lambda(y)$ can be deduced from the spectral intensities as

$$\ell n\, \tau_\lambda(y) = -\int_{x_0(y)}^{x_1(y)} \kappa_\lambda(r)\,\mathrm{d}x = \ell n\left(\frac{I_{1\lambda}(y) - I_{2\lambda}(y)}{I_{0\lambda}}\right) \quad (4)$$

From Eq. (4), the absorption coefficient profile can be estimated from the measured data of transmittance. After that, the source term profile can be deduced from Eq. (3) by using the measured data of outgoing emission radiation intensities. In the cross section of the axisymmetric free flame, the source term profile and the absorption coefficient profile can be represented respectively by polynomials as

$$S_\lambda = \sum_{n=0}^{N} a_n r^n, \qquad \kappa_\lambda = \sum_{m=0}^{M} b_m r^m \quad (5)$$

where a_n and b_m are the coefficients of expansion. Once the source term is available, the temperature distribution can be determined by Eq. (2).

The inverse radiation problem of estimating the unknown absorption coefficient and source term profiles can be formulated as optimization problems. First, we wish to minimize the objective function

$$\Gamma(\boldsymbol{b}) = \sum_{j=1}^{J} \left[\tau_\lambda(y_j; \boldsymbol{b}) - \tau_\lambda^*(y_j)\right]^2 \quad (6)$$

where $\tau_\lambda^*(y_j)$ are the measured transmittance at different position y_j of the line of sight, $\tau_\lambda(y_j; \boldsymbol{b})$ are the estimated transmittance from Eq. (4) for an estimated vector $\boldsymbol{b} = (b_0, b_1, \ldots, b_M)^T$, and J is the number of the line of sight. Finally, we wish to minimize the objective function

$$\Psi(\boldsymbol{a}) = \sum_{j=1}^{J} \left[I_{2\lambda}(y_j; \boldsymbol{a}) - I_{2\lambda}^*(y_j)\right]^2 \quad (7)$$

where $I_{2\lambda}^*(y_j)$ are the measured outgoing radiation intensities at different position y_j and $I_{2\lambda}(y; \boldsymbol{a})$ are the estimated outgoing radiation intensities calculated from Eq. (3) for an estimated vector $\boldsymbol{a} = (a_0, a_1, \ldots, a_N)^T$.

The sensitivity coefficients $\partial I_{2\lambda}/\partial a_n$ and $\partial \ell n\, \tau_\lambda/\partial b_m$ are essential in the solution of inverse problems. To obtain the sensitivity coefficients, we differentiate Eqs. (3) and (4) with respect to a_n and b_m, respectively. A similar numerical quadrature as the direct problem is used for the solution of sensitivity coefficients.

The minimization of the objective function with respect to the desired vector is the most important procedure in solving the minimization problem. In this Note we use the conjugate gradient method to determine the unknown source term and absorption distributions.

Results and Discussion

To examine the effectiveness of the method presented in this Note, several test cases are considered. To demonstrate the effects of measurement errors on the estimation of temperature and absorption coefficient profiles, we consider the random errors. The simulated measured outgoing spectral radiation intensities with random errors are obtained by adding normally distributed errors into these exact data as

$$I_{1\lambda}^*(y_j) = I_{1\lambda}(y_j) + \sigma_{1\lambda,j}\varsigma, \qquad I_{2\lambda}^*(y_j) = I_{2\lambda}(y_j) + \sigma_{2,\lambda,j}\varsigma \quad (8)$$

Here, ς is a normally distributed random variable with zero mean and unit standard deviation. The standard deviations of measured outgoing radiation intensities $\sigma_{1\lambda,j}$ and $\sigma_{2\lambda,j}$ for a γ% measurement error at 99% confidence are determined as

$$\sigma_{1\lambda,j} = \frac{I_{1\lambda}(y_j) \times \gamma\%}{2.576}, \qquad \sigma_{2\lambda,j} = \frac{I_{2\lambda}(y_j) \times \gamma\%}{2.576} \quad (9)$$

where 2.576 arises from the fact that 99% of a normally distributed population is contained within ±2.576 standard deviation of the mean. For the sake of comparison, the rms errors of the estimation for the temperature and absorption coefficient $E_{T,\mathrm{rms}}$ and $E_{\kappa,\mathrm{rms}}$ are defined as

$$E_{T,\mathrm{rms}} = \left\{\frac{1}{R}\int_0^R [T_{\mathrm{estimated}}(r) - T_{\mathrm{exact}}(r)]^2\,\mathrm{d}r\right\}^{\frac{1}{2}} \quad (10a)$$

$$E_{\kappa,\mathrm{rms}} = \left\{\frac{1}{R}\int_0^R [\kappa_{\lambda,\mathrm{estimated}}(r) - \kappa_{\lambda,\mathrm{exact}}(r)]^2\,\mathrm{d}r\right\}^{\frac{1}{2}} \quad (10b)$$

and the averaged values of temperature and absorption coefficient T_{av} and κ_{av} are defined as

$$T_{\mathrm{av}} = \frac{1}{R}\int_0^R S_{\mathrm{exact}}(r)\,\mathrm{d}r, \qquad \kappa_{\mathrm{av}} = \frac{1}{R}\int_0^R \kappa_{\lambda,\mathrm{exact}}(r)\,\mathrm{d}r \quad (11)$$

We consider the source term and absorption coefficient expressed as follows:

$$T(r) = 2500 + 800r - 2600r^2\ \mathrm{K} \quad (12)$$

$$\kappa_\lambda(r) = 1 + 0.2r - r^2\ 1/\mathrm{m} \quad (13)$$

The radius of flame is assumed to 1.0 m in the cross section of measurement, and the working wavelength is selected to be 4.3 μm. In the centerline of the cross section of measurement, the optical thickness is 1.53, and the outgoing spectral emission radiative intensity $I_{2\lambda}(0)$ is 17.6 kW/m^2 μmsr.

We use polynomials of degree 5 to approach the source term and absorption coefficient profiles. There are 10 unknowns so that the number of the line of sight must be larger than 5. In the following analysis seven different positions of the line of sight are selected to measure the outgoing radiative intensities. In the case of $I_{0\lambda} =$ 20 kW/m^2 μmsr, if there are no measurement errors, the error of the temperature estimation $E_{T,\mathrm{rms}}/T_{\mathrm{av}}$ is 0.44% and the error of the absorption coefficient $E_{\kappa,\mathrm{rms}}/\kappa_{\mathrm{av}}$ is 0.004%. The estimated values are very close to the exact values.

The simulated outgoing radiative intensity measurements containing random errors of $\gamma = 1$ and 5% are used to estimate the absorption coefficient and temperature profiles. The results of 15 random samples are shown in Figs. 2 and 3. In the condition of $I_{0\lambda} = 20$ kW/m^2 μmsr, the errors of the temperature estimation are within 2% for the case of $\gamma = 1$% and 11% for the case of $\gamma = 5$%, and the errors of the absorption coefficient estimation are within 6% for the case of $\gamma = 1$% and 28% for the case of $\gamma = 5$%. When the magnitude of measurement errors is increased, the accuracy of the estimation will decrease. Even in the case of $\gamma = 5$%, the reconstruction of temperature is good. However, the estimation of absorption

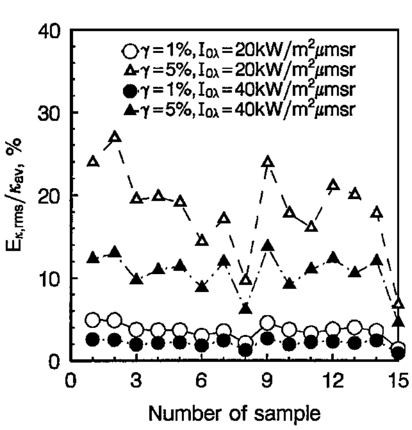

Fig. 2 Errors of estimation of absorption coefficient using noisy input data.

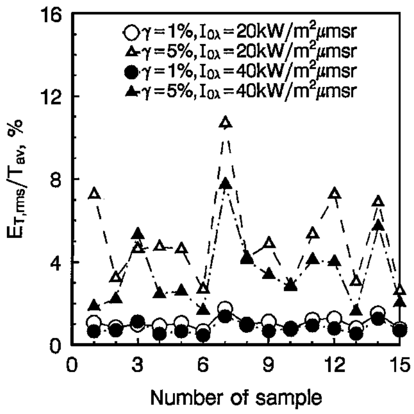

Fig. 3 Errors of estimation of temperature using noisy input data.

coefficient is more sensitive to the measurement errors. By comparing the results of the cases of $I_{0\lambda} = 20$ and 40 kW/m^2 μmsr, it can clearly be seen that increasing the spectral radiative intensity of the laser can improve the accuracy of the reconstruction of absorption coefficient because the accuracy of measured transmittance defined by Eq. (4) will be improved by increasing the spectral radiative intensity of the laser.

For all cases just considered, calculations are started with an initial guesses $\boldsymbol{a}$ and $\boldsymbol{b} = 0$. The CPU time required for each sample of estimation calculation varied from 10 s to 1 min on a personal computer with a 450 MHz Pentium III processor.

Conclusions

An inverse analysis is presented for the estimation of temperature and absorption coefficient profiles for an axisymmetric free flame from the knowledge of the out going spectral emission and transmission radiation intensities. The source term and absorption coefficient profiles are expressed as polynomial-type functions of space ordinate. The inverse problem is formulated in terms of two optimization problems, and the conjugate gradient method is used. Although the temperature and the absorption coefficient are the functions of space variable, the inverse algorithm presented requires measurements of the outgoing radiative intensities only at several different positions of the line of sight. Both exact and noisy input data have been used to test the performance of the proposed method. The effects of measurement errors on the accuracy of the estimations are examined. The results show that the profiles of temperature can be estimated accurately, even with noisy data, but the absorption coefficient is more sensitive to the measurement errors. Increasing the radiative intensity of the laser can improve the accuracy of the absorption coefficient reconstruction.

Acknowledgments

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Mass-Flux and Shock Calculations Through Cryogenic Homogeneous Two-Phase Nozzle Flow

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Nomenclature

$A(\xi)$	= nozzle cross-sectional area
C	= mixture specific heat constant
K	= $1 + R_G X / C$, parameter replacing γ in homogeneous two-phase flow
$\dot{m}$	= mass-flow rate
N	= linearity parameter in the Henry–Fauske flow quality derivative[1]
p	= pressure
R_G	= vapor-phase gas constant
T	= local mixture temperature
X	= flow quality
α	= void fraction
γ	= vapor-phase specific heat ratio
v	= specific volume
ξ	= axial distance
ρ	= mixture density

Subscripts

e	= equilibrium
G	= vapor phase
L	= liquid phase
t	= total quantity
th	= throat location
1	= upstream of shock
2	= downstream of shock

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